Computability, Complexity, and Randomness 2026
University of Leeds School of Mathematics
Leeds, UK
07 – 11 September 2026

https://www.computability.org/ccr2026/

All activities are in the MALL on Level 8 in the School of Mathematics.

Monday, 07 September 2026

Before 10:00am:	Coffee available!
10:00am – 10:50am:	Liling Ko (Nanyang Technological University) *Embedding distributive lattices in the punctual degrees of the rationals*
10:50am – 11:10am:	Morning break!
11:10am – 12:00pm:	Raphaël Carroy (Università degli Studi di Torino) *Reduction and games in topology*
12:00pm – 2:00pm:	Lunch!
2:00pm – 2:30pm:	Giorgio Genovesi (University of Leeds) *Bisimulations, the tree interpretation, and transfinite recursion*
2:30pm – 3:00pm:	Afternoon break!
3:00pm – 3:30pm:	Luca Facchinetti (Universitá degli Studi di Udine) *New results about reverse mathematics and wqo definitions*
3:40pm – 4:10pm:	Benjamin Koch (Swansea University) *The computational strength of high-dimensional points*
4:20pm – 4:50pm:	Ellen Hammatt (Technische Universität Wien) *κ-Turing degrees and κ-computable approximations*

Tuesday, 08 September 2026

Before 10:00am:	Coffee available!
10:00am – 10:50am:	Quentin Le Houérou (Université Paris-Est Créteil) *Ramsey-like theorems for separable permutations*
10:50am – 11:10am:	Morning break!
11:10am – 12:00pm:	Oriola Gjetaj (Technische Universität Wien) *Ramsey-type principles for barriers and more*
12:00pm – 2:00pm:	Lunch!
2:00pm – 2:30pm:	Orazio Nicolosi (Università degli Studi di Torino) *A game characterization for weak continuous reducibility*
2:30pm – 3:00pm:	Afternoon break!
3:00pm – 3:30pm:	Dino Rossegger (Technische Universität Wien) *A topological highness notion*
3:40pm – 4:10pm:	Georgii Potapov (Royal Holloway, University of London) *Randomness deficiency with respect to a class*
4:30pm – 6:00pm	Reception!

Wednesday, 09 September 2026

Before 10:00am:	Coffee available!
10:00am – 10:50am:	Jack Lutz (Iowa State University) *The recent advent of multi-head finite-state dimensions*
10:50am – 11:10am:	Morning break!
11:10am – 12:00pm:	Yiping Miao (University of California, Berkeley) *Dimensions of generic and random reals*
12:00pm onward:	Free afternoon!

Thursday, 10 September 2026

Before 10:00am:	Coffee available!
10:00am – 10:50am:	Isabella Scott (University of Wisconsin–Madison) *Comparing presentations of Polish spaces*
10:50am – 11:10am:	Morning break!
11:10am – 12:00pm:	Henry Klatt (George Washington University) *Cohesive Products of Galois Extensions*
12:00pm – 2:00pm:	Lunch!
2:00pm – 2:30pm:	Antonio Nakid Cordero (Technische Universität Wien) *Effectively closed sets and enumeration reducibility*
2:30pm – 3:00pm:	Afternoon break!
3:00pm – 3:30pm:	Heer Tern Koh (University of Electronic Science and Technology of China) *On a reduction on compact Polish spaces*
3:40pm – 4:10pm:	Daniel Mourad (Nanjing University) *Escaping continuity within α-many steps*
4:20pm – 4:50pm:	Margarita Reyes García (Universidad Nacional Autónoma de México) *Approximating Kolmogorov complexity in fractal graphs using BDM: A methodological proposal*

Friday, 11 September 2026

Before 10:00am:	Coffee available!
10:00am – 10:50am:	Keita Yokoyama (Tohoku University) *Nonstandard arguments in reverse mathematics* *(in memory of Kazuyuki Tanaka)*
10:50am – 11:10am:	Morning break!
11:10am – 12:00pm:	Rod Downey (Victoria University of Wellington) *Yet more on Q- and Z- reducibilities*
12:00pm – 2:00pm:	Lunch!
2:00pm – 2:30pm:	Yudai Suzuki (National Institute of Technology, Oyama College) *Generalizations of the second incompleteness theorem and its surroundings*
2:30pm – 3:00pm:	Afternoon break!
3:00pm – 3:30pm:	Kenshi Miyabe (Meiji University) *Decomposing point-to-set principles: Coding and effectivity*
3:40pm – 4:10pm:	Yuzuki Kaneko (Tohoku University) *Reverse mathematics of the classification of maximal filter spaces*
4:20pm – 5:10pm:	Iqra Altaf (University of California, Los Angeles) *online talk* *Distance sets bounds for polyhedral norms via effective dimension*

Abstracts (alphabetical by author)

Iqra Altaf (University of California, Los Angeles)

Distance sets bounds for polyhedral norms via effective dimension

We prove that, for every polyhedral norm on $\mathbb{R}^d$ and every set $E \subseteq \mathbb{R}^d$ of Hausdorff (packing) dimension s, the Hausdorff (packing) dimension of the distance set of E with respect to that norm is at least $s - (d-1)\ ({}^{s}/{}_{d})$.

Raphaël Carroy (Università degli Studi di Torino)

Reduction and games in topology

Inspired by computability-theoretic reduction, William W. Wadge began looking at topological reductions in the early 1970's. He made a breakthrough by using the determinacy of an infinite two-player game with perfect information, now called the Wadge game. This launched a whole line of research in topology, and more specifically in descriptive set theory. I will give a brief and partial overview of past and present results in this line of research.

Rod Downey (Victoria University of Wellington)

Yet more on Q- and Z- reducibilities

We discuss the two reducibilities above. We look at where they come from, and mention how they seem tied up with particularly group theory and Henkin-type constructions. We will also look at some recent structural results. There are many many open questions here.

Luca Facchinetti (Universitá degli Studi di Udine)

New results about reverse mathematics and wqo definitions

Joint work with Alberto Marcone

The term "reverse mathematics" refers to a broad research program in the foundations of mathematics, whose main goal is to find the minimal axioms needed to prove the theorems of ordinary mathematics. One topic of interest in reverse mathematics is the theory of *well-quasi-orders* ([1]).

Definition 1. A quasi-order is a pair $(Q, \preceq)$ where Q is a set and $\preceq$ is a binary relation on Q which is reflexive and transitive.

The notion of well-quasi-order (or wqo for short) has many equivalent definitions.

Theorem 2. *Let $(X, \leq)$ be a quasi-order. The following are equivalent:*

(w1) for every infinite sequence $(x_n)_{n\in\omega}$ of elements of Q, there exist $n < m$ such that $x_n \preceq x_m$;

(w2) $(X, \leq)$ has no infinite descending sequences and no infinite antichains;

(w3) for every sequence $(x_n)_{n\in\omega}$ there is $A \subseteq \mathbb{N}$ infinite such that for every $n, m \in A$ with $n < m$ we have $x_n \leq x_m$;

(w4) *for every* $Y \subseteq X$ *there is* $Z \subseteq Y$ *finite such that for every* $y \in Y$ *there is* $z \in Z$ *with* $z \leq y$*;*

(w5) *there is no infinite sequence of upward closed sets of* X *which is strictly increasing with respect to inclusion;*

(w6) *there is no infinite sequence of downward closed sets of* X *which is strictly decreasing with respect to inclusion;*

(w7) *every linear extension of* $\leq$ *is a well order.*

In 2005, Cholak, Marcone and Solomon [2] studied the computational strength of the equivalences contained in Theorem 2. Their work mostly concerns the distinction between definitions that are equivalent in the base theory of reverse mathematics RCA_0 (they turn out to be (w1), (w4), (w5) and (w6)), and those which are not.

In particular, they proved the following result.

Theorem 3. *The implications between (w1), (w2),* (w3) *and* (w7) *which are provable in* RCA_0 *are exactly the ones in the transitive closure of the diagram:*

$$\begin{tikzcd} & & (w2) \\ (w3) \arrow[r] & (w1) \arrow[ur] \arrow[dr] & \\ & & (w7) \end{tikzcd}$$

Our aim is revisiting that article and refining the results.

We improved Theorem 3 by adding two new definitions. The first one is the conjunction $(w2) \wedge (w7)$.

Theorem 4. $(w2) \wedge (w7) \rightarrow (w1)$ *follows from either* WKL_0 *or* ADS*, but it is not provable within* WWKL_0*.*

Another equivalent definition of wqo, which appears in [3] and we call (wD) is: "$\P(X)$ is well-founded with respect to $\preceq^\flat$", where $\preceq^\flat$ is defined by

$$X \preceq^\flat Y \iff \forall x \in X \, \exists y \in Y \; x \preceq y.$$

It is easy to see that (wD) implies $(w1)$ within RCA_0.

Theorem 5. *Over* RCA_0*, the implication* $(w1) \rightarrow (wD)$ *is equivalent to* ACA_0*.*

Theorem 6. *Over* RCA_0*, the implication* $(wD) \rightarrow (w3)$ *is equivalent to* $\mathsf{RT}^1_{<\infty}$*.*

Theorem 7. *Over* ADS*, the implication* $(w3) \rightarrow (wD)$ *is equivalent to* ACA_0*.*

These results are summarized in the following.

Corollary 8. *The implications between* (wD), $(w1)$, $(w2)$, $(w3)$, $(w7)$ *and* $(w2) \wedge (w7)$ *which are provable in* RCA_0 *are exactly the ones in the transitive closure of the diagram:*

$$\begin{tikzcd} (wD) \arrow[dr] & & & (w2) \\ & (w1) \arrow[r] & (w2) \wedge (w7) \arrow[ur] \arrow[dr] & \\ (w3) \arrow[ur] & & & (w7) \end{tikzcd}$$

A few questions are still open:

Question 9. Are implications $(w7) \rightarrow (w1)$ and $(w7) \rightarrow (w2)$ equivalent to WKL_0 over RCA_0?

Question 10. Is the implication $(w3) \rightarrow (wD)$ equivalent to ACA_0 over RCA_0?

[1] Alberto Marcone, *The reverse mathematics of wqos and bqos*, Well-quasi orders in computation, logic, language and reasoning—a unifying concept of proof theory, automata theory, formal languages and descriptive set theory, Trends Log. Stud. Log. Libr., vol. 53, Springer, Cham, [2020] ©2020, pp. 189–219.

[2] Peter Cholak, Alberto Marcone, and Reed Solomon, *Reverse mathematics and the equivalence of definitions for well and better quasi-orders*, J. Symbolic Logic **69** (2004), no. 3, 683–712.

[3] Reinhard Diestel, *Relating subsets of a poset, and a partition theorem for WQOs*, Order **18** (2001), no. 3, 275–279.

Giorgio Genovesi (University of Leeds)

Bisimulations, the tree interpretation, and transfinite recursion

In the reverse mathematics of second order arithmetic, the statement that every well-founded relation has a bisimulation is equivalent to arithmetic transfinite recursion. The usual proof goes through the comparability of well orders, which makes use of the Kleene normal form. In joint work with Emanuele Frittaion, we show that such results generalize to uncountable settings where the Kleene normal form may not hold. In particular, over a weak theory of sets, the existence of bisimulations is equivalent to elementary transfinite recursion. In this talk I will sketch the proof of this result and go over some adjacent results and how they relate to interpretations between systems of arithmetic and fragments of set theory without the powerset axiom.

Oriola Gjetaj (Technische Universität Wien)

Ramsey-type principles for barriers and more

In this talk, we will analyze generalizations of Friedman's Free Set and Thin Set theorems, as well as the Rainbow Ramsey Theorem, to colorings of barriers. I will give a general overview of the background and the principles we are interested in, then introduce our generalized principles. Finally, we will analyze the computability of these principles and how they compare with existing results.

This is joint work with Lorenzo Carlucci.

Ellen Hammatt (Technische Universität Wien)

κ-Turing degrees and κ-computable approximations

We study the generalisation of computations over admissible ordinals κ. Rather than using classical κ-reduction we define a reducibility which we call κ-Turing by using functionals that are Σ_1-definable over L_κ. We consider this reduction on functions $f : \rho \to \rho$, where ρ is the Σ_1-projectum of κ. We note that this generalises the definition made for $\kappa = \omega_1^{ck}$ by Bienvenu, Greenberg and Monin. For non-projectable κ, these reducibilities coincide, however in the projectable setting κ-Turing is a new degree structure. In fact, for projectable κ with regular Σ_1-projectum the situation is dramatically different. We study the κ-Turing in the setting of a generalisation of the transfinite hierarchy of Turing degrees based on the mind-changes of computable approximations introduced by Downey and Greenberg.

This is joint work with Juan Aguilera and Noam Greenberg.

Yuzuki Kaneko (Tohoku University)

Reverse mathematics of the classification of maximal filter spaces

A topological space is called a poset space when it is formed from filters on a partially ordered set. Poset spaces are T_0 spaces and not necessarily metrizable. In particular, we use the spaces of the following filters.

Definition 1. Let $(P,<)$ be a partially ordered set and let F be a filter on $(P,<)$.

- F is called an unbounded filter when $\forall p \in P\, \exists r \in F\, (p \nleq r)$.
- F is called a maximal filter when there is no filter G on $(P,<)$ such that $F \subsetneq G$.

Every maximal filter is an unbounded filter.

When we consider only second-countable spaces, the space of unbounded filters (UF space) has several equivalent representations, such as spaces of non-principal filters (NP spaces), quasi-Polish spaces, or $\mathbf{\Pi}^0_2$ subspaces of the countable power of the Sierpiński space [2, 4]. Moreover, every second-countable T_0 space is embeddable as a subspace of a UF space. Therefore, considering subspaces of UF spaces enables the analysis of more general topological objects. Since second-countable poset spaces in the above sense can be formalized in second-order arithmetic, the logical strengths of several theorems, including Urysohn's metrization theorem, have been analyzed in the context of reverse mathematics. Indeed, Π^1_1-CA_0 proves "every regular UF space is completely metrizable", while "every regular space of maximal filters (MF space) is completely metrizable" is equivalent to Π^1_2-CA_0 over Π^1_1-CA_0 [5]. In this talk, we proceed with the study of poset spaces, and analyze the strength of classification theorems for poset spaces in the context of reverse mathematics.

Urysohn's theorem gives a necessary and sufficient condition for MF spaces to be Polish spaces. Then, when is an MF space quasi-Polish? In general, can we classify MF spaces? For quasi-Polish spaces (hence for UF spaces), the following characterization is known.

Theorem 2 (generalized Hurewicz theorem [3, de Brecht Thm 7.2])**.** *Let Y be a $\mathbf{\Pi}^1_1$ subspace of a quasi-Polish space (X,d). Then Y either is a quasi-Polish space or contains a $\mathbf{\Pi}^0_2$ subspace of X homeomorphic to S_0, S_D, S_1, or $\mathbb{Q}$.*

Since every MF space is a $\mathbf{\Pi}^1_1$ subspace of a UF space, we can apply the above theorem to classify MF spaces. More precisely, we can show that an MF space is homeomorphic to a quasi-Polish space if and only if it does not contain a $\mathbf{\Pi}^0_2$ subspace homeomorphic to the rationals $\mathbb{Q}$. Indeed, this dichotomy depends on whether that MF space is completely Baire or not.

Furthermore, this analysis can be formalized within second-order arithmetic. For UF spaces, it is known that "every UF space is homeomorphic to a quasi-Polish space" is equivalent to Π^1_1-CA_0 over RCA_0 [4]. As our main results, we determine the exact logical strength of the above Hurewicz-like classifications.

Theorem 3. *Over Π^1_1-CA_0, the following statements are equivalent:*

1. Π^1_2-CA_0.
2. *Every $\mathbf{\Pi}^1_1$ subspace $Y \subseteq X$ of a quasi-Polish space (X,d) is a quasi-Polish space when both (Y,d) and $(Y,\hat{d})$ are completely Baire.*
3. *Every completely Baire $\mathbf{\Pi}^1_1$ subspace $Y \subseteq X$ of a Polish space (X,d) is a Polish space.*
4. *Every completely Baire MF space is homeomorphic to a quasi-Polish space.*

Here, $(X,\hat{d})$ is a Polish space that admits a natural Σ^0_2-open bijection from (X,d).

On the other hand, the characterization of the "non-completely Baire" case is slightly weaker.

Theorem 4. *Over Π^1_1-CA_0, the following statements are equivalent:*

1. Σ^1_2-DC_0.
2. *Every $\mathbf{\Pi}^1_1$ subspace $Y \subseteq X$ of a quasi-Polish space (X, d) contains a $\mathbf{\Pi}^0_2$ subspace of Y homeomorphic to S_0, S_D, S_1, or $\mathbb{Q}$ when either (Y, d) or $(Y, \hat{d})$ is not completely Baire.*
3. *Every $\mathbf{\Pi}^1_1$ subspace $Y \subseteq X$ of a Polish space (X, d) that is not completely Baire contains a closed set homeomorphic to $\mathbb{Q}$.*
4. *Every MF space that is not completely Baire contains a $\mathbf{\Pi}^0_2$ subspace homeomorphic to $\mathbb{Q}$.*

Furthermore, extending our analysis to the projective hierarchy, we obtain the following equivalence.

Theorem 5. *Over Π^1_1-$\mathsf{CA}_0 + \mathbf{V} = \mathbf{L}$, the following statements are equivalent for each $n \geq 1$:*

1. Σ^1_{n+1}-DC_0.
2. *Every $\mathbf{\Pi}^1_n$ subspace $Y \subseteq X$ of a Polish space (X, d) that is not completely Baire contains a closed subset homeomorphic to $\mathbb{Q}$.*

While DC is well-known to be equivalent to the Baire category theorem for complete metric spaces within ZF [1], this result provides a much finer analysis of this relationship within the framework of second-order arithmetic.

This is joint work with Keita Yokoyama.

[1] Charles E. Blair. The Baire category theorem implies the principle of dependent choices. *Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys.*, 25(10):933–934, 1977.

[2] Matthew de Brecht. Quasi-Polish spaces. *Ann. Pure Appl. Logic*, 164(3):356–381, 2013.

[3] Matthew de Brecht. A generalization of a theorem of Hurewicz for quasi-Polish spaces. *Logical Methods in Computer Science*, 14(1), 2018.

[4] Y. Kaneko and K. Yokoyama, Quasi-Polish spaces and spaces of filters in second-order arithmetic, preprint, arXiv:2605.15052 [math.LO], 2026.

[5] Carl Mummert. *On the reverse mathematics of general topology.* The Pennsylvania State University, 2005.

Henry Klatt (George Washington University)

Cohesive products of Galois extensions

Cohesive products are a computability theoretic analog to the ultraproduct. The domain is restricted to partial computable functions, and the ultrafilter is replaced with a cohesive set, which cannot be split into infinite pieces by any computably enumerable set. In this talk, we will present an introduction to the topic, and then discuss recent work by R. Dimitrov, V. Harizanov, H. Klatt, and K. Srinivasan on cohesive products of Galois extensions. In particular, we will focus on the cohesive product of Galois actions, which leads one naturally into an exciting world of non-standard number theory, complete with pseudo-finite Galois extensions and infinite polynomials.

Liling Ko (Nanyang Technological University)

Embedding distributive lattices in the punctual degrees of the rationals

We study the structure $\mathcal{P}(\mathbb{Q})$, the punctual degrees of the rationals, where two primitive recursive copies $\mathcal{A}$ and $\mathcal{B}$ of $\mathbb{Q}$ are defined as belonging to the same degree in $\mathcal{P}(\mathbb{Q})$ if there exists a primitive recursive map from $\mathcal{A}$ to $\mathcal{B}$ and also a primitive recursive map from $\mathcal{B}$ to $\mathcal{A}$. It is known that $\mathcal{P}(\mathbb{Q})$ has a maximal degree $\mathcal{T}$ [Melnikov, Ng 2019], and the diamond lattice can be embedded in $\mathcal{P}(\mathbb{Q})$ [Dorzhieva, Hammatt 2025]. Our main result is that we can embed the countable atomless boolean algebra in $\mathcal{P}(\mathbb{Q})$. As a corollary, every finite distributive lattice can be embedded in $\mathcal{P}(\mathbb{Q})$. Furthermore, we also show that the top element of the diamond lattice and the top element of the distributive lattice generated by three atoms can $\mathcal{T}$ itself.

This is joint work with Ellen Hammatt and Heer Tern Koh.

Benjamin Koch (Swansea University)

The computational strength of high-dimensional points

Over the last decade, algorithmic information theory has contributed to several advances in geometric measure theory. This work has largely stemmed from the point-to-set principle of J. Lutz and N. Lutz [1], who showed that the classical Hausdorff and packing dimensions of a subset of $\mathbb{R}^n$ is determined by the Kolmogorov complexity of its points. In particular, if a set $E \subseteq \mathbb{R}^n$ has Hausdorff dimension s, then for all oracles $A \in 2^{\mathbb{N}}$ and all $\varepsilon > 0$, there exists $x \in E$ such that

$$\mathrm{eDim}^A(x) := \liminf_{n\to\infty} \frac{K^A_{2^{-n}}(x)}{n} > s - \varepsilon,$$

where $K^A_{2^{-n}}(x)$ is the complexity of a rational approximation of x to precision 2^{-n}.

In this talk, we investigate the computational power of such dimension-approximating points from the perspective of the Weihrauch degrees. We formulate the problem DAC of dimension-approximating choice on closed subsets of $2^{\mathbb{N}}$, as well as the problem DWC of dimension-witnessing choice, the latter of which allows us to consider those sets which actually contain a point whose effective dimension relative to some oracle equals the set's Hausdorff dimension. We show that this latter problem is equivalent to $\mathsf{C}_{\mathbb{N}^{\mathbb{N}}}$, choice on Baire space, while the former is above both choice on the reals and RT^1_2.

These results are from joint work in progress with Manlio Valenti.

[1] Jack H. Lutz and Neil Lutz, "Algorithmic information, plane Kakeya sets, and conditional dimension", ACM Transactions on Computation Theory, vol. 10 (2018), no. 2, pp. 7:1–7:22

Heer Tern Koh (University of Electronic Science and Technology of China)

On a reduction on compact Polish spaces

An emerging direction in computable structure theory is the study of the effective content of categoricity problems of Polish spaces. Here, (computable) Polish spaces are studied up to computable homeomorphisms. However, as the inverse of a type two computable function is not necessarily computable, in particular, the inverse of a computable homeomorphism need not be computable, one can instead consider the relation "being computably homeomorphic" as a reduction. More formally, a *computable Polish space* is a tuple $\overline{(\{\alpha_i\}_{i\in\omega}, d)}$ such that $d(\alpha_i, \alpha_j)$ is a real uniformly computable in i, j. For a fixed homeomorphism type X, we may then define $A \leq_{ct} B$ where A, B are computable Polish spaces homeomorphic to X if there exists a computable homeomorphism from B to A. The collection of all degrees induced by the reduction $\leq_{ct}$ is then termed the *computable topological degrees of* X and denoted as $CT(X)$.

Recent work by N. Greenberg, A. Melinkov, I. Scott, and D. Turetsky studied the computable topological degrees of Stone spaces, investigating how degree-theoretic properties of $CT(X)$ relates to structural properties of X itself. In this talk, we prove a meta-theorem, providing a sufficient condition for the computable topological degrees of a Polish space X to admit an embedding of the c.e. Turing degrees. As an application of our techniques, we also prove sufficient conditions for when the computable topological degrees fail to form a lattice, and when the analogue of the Cantor-Schroder-Bernstein Theorem fails.

Quentin Le Houérou (Université Paris-Est Créteil)

Ramsey-like theorems for separable permutations

Following the discovery that the infinite Ramsey theorem for pairs and two colors, RT^2_2, escapes the structural phenomenon known in reverse mathematics as the "Big Five", many Ramsey-like statements have been studied from computational and reverse-mathematical perspectives.

In this talk, we consider weakenings of RT^2_2 where the graphs under consideration avoid certain patterns. Among these patterns, we show that separable permutations, a well-studied class in combinatorics, play a central role.

In particular, we prove that, for every separable permutation p, the statement $\mathsf{coRT}^2_2(p)$, asserting that every infinite p-avoiding graph has an infinite clique or anti-clique, admits probabilistic solutions, and that $\emptyset'$-DNC degrees are precisely the strength needed to compute them.

This is a joint work with Ludovic Patey.

Jack Lutz (Iowa State University)

The recent advent of multi-head finite-state dimensions

Finite-state devices may be enhanced by adding trailing read heads ("probes to the past") in order to augment their finite memories with limited detection of long-range correlations. This was done long ago [5] for finite automata tasked with recognizing sets of finite strings and very recently [2, 3, 6, 7] for the finite-state gamblers and compressors used to define finite-state versions of the Hausdorff and packing dimensions of infinite sequences over finite alphabets [1, 4]. Intuitively, these dimensions are, respectively, the lower and upper asymptotic *densities*, or *rates* of information in these sequences, *as perceived by* the corresponding classes of devices. This talk will discuss these recent developments and their future prospects.

[1] Krishna B. Athreya, John M. Hitchcock, Jack H. Lutz, and Elvira Mayordomo, *Effective strong dimension in algorithmic information and computational complexity*, SIAM Journal on Computing **37** (2007), no. 3, 671–705.

[2] Julianne Cruz, Sho Glashausser, Xiaoyuan Li, and Neil Lutz, *Adaptive multi-head finite-state gamblers*, Timeless machines: Computability across eras — 22nd conference on Computability in Europe, CiE 2026, Trier, Germany, July 27-31, 2026, proceedings, 2026, pp. 224–239.

[3] Julianne Cruz, Sho Glashausser, and Neil Lutz, *One adaptive trailing head can outperform many oblivious trailing heads*, Machines, computations, and universality (MCU 2026), to appear.

[4] Jack J. Dai, James I. Lathrop, Jack H. Lutz, and Elvira Mayordomo, *Finite-state dimension*, Theoretical Computer Science **310** (2004), no. 1, 1–33.

[5] Markus Holzer, Martin Kutrib, and Andreas Malcher, *Multi-head finite automata: Characterizations, concepts and open problems*, Electronic Proceedings in Theoretical Computer Science (EPTCS), 2009, pp. 93–107.

[6] Xiang Huang, Xiaoyuan Li, Jack H. Lutz, and Neil Lutz, *Multi-head finite-state dimension*, 51st International Symposium on Mathematical Foundations of Computer Science (MFCS 2026), 2026, pp. 59:1–59:14.

[7] Neil Lutz, *Multi-head finite-state compression*, Latin 2026: Theoretical informatics, to appear.

Yiping Miao (University of California, Berkeley)

Dimensions of generic and random reals

Measure and category are two orthogonal notions. A typical element in a comeager set is generic, and a typical element in a conull set is random. There exists a comeager null set, and the set of (Cohen) generic reals provides an example. The set also has Hausdorff dimension zero. However, the set has a perfect subset, suggesting largeness under certain measures. We consider an extended notion of Hausdorff dimension, using a gauge function to count how much an open ball contributes.

For any set of reals, the gauge profile of it is the set of gauge functions making the set measure positive. We will show that the complexity of the dense sets the generic meets corresponds nicely with the complexity of functions generating its gauge profile. Then we look at Sacks generics and Mathias generics. These two examples provide a comparison between gauge profile and the complexity of the collection of dense sets.

We also looked into the gauge profile of the set of reals compressible by $1/2$. This is the random real analogue of the (Cohen) generic case. The set is known to have dimension $1/2$ and our work shows the gauge profile is supported on its dimension.

Kenshi Miyabe (Meiji University)

Decomposing point-to-set principles: Coding and effectivity

Point-to-set principles connect classical fractal dimensions of sets with pointwise algorithmic dimensions. For example, the point-to-set principle of [Lutz and Lutz(2018)] expresses the Hausdorff and packing dimensions of a nonempty set $E \subseteq \mathbb{R}^n$ as

$$\dim_{\mathrm{H}}(E) = \min_{A \subseteq \mathbb{N}} \sup_{x \in E} \liminf_{r \to \infty} \frac{K_r^A(x)}{r}, \qquad \dim_{\mathrm{P}}(E) = \min_{A \subseteq \mathbb{N}} \sup_{x \in E} \limsup_{r \to \infty} \frac{K_r^A(x)}{r}.$$

Here $K_r^A(x)$ is the Kolmogorov complexity, relative to an oracle A, of describing x to precision 2^{-r}; explicitly,

$$K_r^A(x) = \min\{K^A(q) : q \in \mathbb{Q}^n, \|q - x\| \leq 2^{-r}\},$$

where $K^A(q)$ denotes the prefix-free Kolmogorov complexity of a rational point q relative to A.

We show that this connection can be decomposed into two independent steps, separating a classical coding principle from the passage to effectivity. For Hausdorff dimension on $\mathbb{R}^n$, the situation is summarized in Table 1.

	Cover/test	Complexity	Measure
Classical	Definition	Ball codes (contribution)	Weak duality principle of [Cutler(1995)]
Effective		[Lutz and Mayordomo(2008)]	[Lutz and Lutz(2025)]

Table 1: Classical and effective formulations of Hausdorff dimension on $\mathbb{R}^n$.

On Cantor space, partial randomness yields an effective test characterization [Downey and Hirschfeldt(2010), Prop. 13.5.3], and characterizations by gales on cylinder sets have been studied extensively, whereas in Euclidean space a formulation in terms of the local mass of measures is more natural.

On sequence spaces, coding-theoretic characterizations of Hausdorff dimension go back to [Ryabko(1986)], who identified the optimal worst-case asymptotic compression ratio of a combinatorial source with its Hausdorff dimension. Ball codes may be viewed as a Euclidean, ball-based counterpart of this classical coding principle.

For packing dimension, [Athreya et al.(2007)Athreya, Hitchcock, Lutz, and Mayordomo] established the gale and complexity characterizations on Cantor space. The corresponding notions on $\mathbb{R}^n$ were subsequently introduced and used in the point-to-set principle.

The point-to-set principle connects the classical cover/test formulation with the effective complexity formulation. We argue that this connection can be understood as the composition of two conceptually distinct transitions:

$$\text{cover/test} \longrightarrow \text{coding/complexity}, \qquad \text{classical} \longrightarrow \text{effective}.$$

The first replaces geometric covers by coding complexity, whereas the second passes from classical to effective objects.

For the first transition, we introduce *ball codes.* A ball code is a map

$$\varphi\colon D \to \{\text{closed balls in } \mathbb{R}^n\},$$

where $D \subseteq 2^{<\omega}$ is prefix-free, so that Kraft's inequality applies. For $x \in \mathbb{R}^n$ and precision $r \in \omega$, let

$$K_\varphi(x;r) = \min\{|p| : p \in D,\ x \in \varphi(p),\ \operatorname{diam}(\varphi(p)) \leq 2^{-r}\}.$$

Thus $K_\varphi(x;r)$ is the length of a shortest name of a sufficiently small ball containing x (with $\min \emptyset = \infty$). No computability requirement is imposed on φ.

We prove that, for every nonempty $E \subseteq \mathbb{R}^n$,

$$\dim_{\mathrm{H}}(E) = \min_{\varphi} \sup_{x \in E} \liminf_{r \to \infty} \frac{K_\varphi(x;r)}{r}, \qquad \dim_{\mathrm{P}}(E) = \min_{\varphi} \sup_{x \in E} \limsup_{r \to \infty} \frac{K_\varphi(x;r)}{r},$$

where φ ranges over all prefix-free ball codes. In both formulas the minimum is attained. These are purely classical statements. In particular, the appearance of a pointwise description rate is not in itself an effective phenomenon.

The usual point-to-set principle is then recovered by effectivization. A ball code can be replaced by a countable rational codebook, encoded in an oracle, and simulated by an optimal oracle prefix-free machine with constant overhead without changing the asymptotic rates. Thus the geometric content (both the pointwise rate and the minimization over codes) is classical; the oracle provides an effective representation of a code that is optimal for E.

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Daniel Mourad (Nanjing University)

Escaping continuity within α-many steps

In the context of reverse mathematics and computable analysis, continuity of a multivalued function (also known as a problem) $P :\subset \mathbb{N}^\mathbb{N} \rightrightarrows \mathbb{N}^\mathbb{N}$ is defined by the existence of a continuous *global* realizer of P. However, every student of topology is familiar with the following slogan: "continuity is a *local* property!"

We classify some ways in which continuity can fail locally. The story begins with a lemma used by Pauly and Soldá [2] to characterize $\mathsf{ACC}_\mathbb{N}$ as the $\leq^*_\mathrm{W}$-least first order problem.

Lemma 1 ([2])**.** *Let $P :\subset \mathbb{N}^\mathbb{N} \rightrightarrows \mathbb{N}^\mathbb{N}$ be a multivalued function. Suppose that there is a convergent sequence of points $x_1, x_2, \cdots \to x \in \mathbb{N}^\mathbb{N}$ such that $P|_{\{x\}\cup\{x_i : i\in\mathbb{N}\}}$ is not continuous. Then, for each $y \in P(x)$ there is $\rho \sqsubset y$ such that for all $\sigma \sqsubset x$ there is $x_i \sqsupset \sigma$ such that $P(x_i) \cap [\![\rho]\!] = \emptyset$.*

In Lemma 1, the point x is the only local discontinuity of $P|_{\{x\}\cup\{x_i:i\in\mathbb{N}\}}$. Suppose that we are reading x while writing down y in an attempt to solve P. As we do this, we will eventually write down some initial segment $\rho \sqsubset y$ that can be diagonalized against by some x_i which diverges from x is after what we have already read. This is a specific case of a more general phenomenon which we describe in Theorem 2.

Theorem 2. *Let $P :\subset \mathbb{N}^\mathbb{N} \rightrightarrows \mathbb{N}^\mathbb{N}$ be a multivalued function and let $x \in \mathrm{dom}(P)$. The following are equivalent.*

1. *P is locally discontinuous at x; that is, there is no open set $U \subset \mathbb{N}^\mathbb{N}$ with $x \in U$ such that $P|_U$ is continuous.*
2. *For each $y \in P(x)$ there is $\rho \sqsubset y$ such that for all $\sigma \sqsubset x$ there is τ with $\sigma \sqsubset \tau \not\sqsubset x$ such that $P|\,[[\![\tau]\!] \times [\![\rho]\!]]$ is not continuous. In symbols,*

$$(\forall y \in P(x))(\exists \rho \sqsubset y)(\forall \sigma \sqsubset x)(\exists \tau)(\sigma \sqsubset \tau \not\sqsubset x \wedge P|\,[[\![\tau]\!] \times [\![\rho]\!]] \text{ is not continuous})$$

Here $P|[[\![\sigma]\!] \times [\![\rho]\!]]$ denotes the restriction of P to instances in the cone $[\![\sigma]\!]$ and solutions in the cone $[\![\rho]\!]$. Note that we allow for instances to not have solutions.

In Theorem 2, we do not always obtain a full diagonalization against ρ. Instead, we gain the understanding that no matter how much of x we read, there will always be an extension such that we have written too much of y to continuously solve P above what we have already read (σ) and written (ρ). Every discontinuous problem has a local discontinuity, so we can iterate this process on a local discontinuity of $P|[[\![\sigma]\!] \times [\![\tau]\!]]$. By counting how many times we can do this before running into an instance with no solution above $[\![\rho]\!]$, we obtain a transfinite ordinal ranking of discontinuity.

Definition 3. Let $P :\subset \mathbb{N}^\mathbb{N} \rightrightarrows \mathbb{N}^\mathbb{N}$ be a problem and let $x \in \mathrm{dom}(P)$ be a local discontinuity of P. We assign a countable ordinal rank of discontinuity to x as follows.

- If $P(x)$ is empty then we say that x is a rank-0 discontinuity of P.
- For countable $\alpha > 0$, x is a rank-α discontinuity of P if α is the least ordinal such that for all $y \in P(x)$ there is $\rho \sqsubset y$ such that for all $\sigma \sqsubset x$ there exists $x' \in \mathrm{dom}(P) \cap [\![\sigma]\!]$ with $x' \neq x$ such that x' is a rank-β discontinuity of $P|[[\![\sigma]\!] \times [\![\rho]\!]]$ for some $\beta < \alpha$.
- If there is no such ordinal α then we say that x is a rank-∞ discontinuity of P.

The similarity of Definition 3 to Cantor-Bendixson rank is not by chance. A problem P having a discontinuity of rank $\leq \alpha$ is equivalent to P being discontinuous on a set $A \subset \mathbb{N}^\mathbb{N}$ whose points all have Cantor-Bendixson rank $\leq \alpha$. Furthermore, we can characterize Definition 3 by a two player discontinuity game, which we will define in the talk. Finally, we show that there is a $\leq^*_{\mathrm{W}}$-least discontinuous problem which has a discontinuity of rank $\leq \alpha$; namely, the problem of solving $\mathsf{ACC}_{\mathbb{N}}$ in at most α many attempts, in the sense that a new instance of $\mathsf{ACC}_{\mathbb{N}}$ is given for each attempt. We refer to this problem as $\mathsf{ACC}^{\alpha}_{\mathbb{N}} :\subset (\mathbb{N} \cup \{\#\})^{\mathbb{N}} \rightrightarrows (\mathbb{N} \cup \{\#\})^{\mathbb{N}}$. The sole local discontinuity of $\mathsf{ACC}^{\alpha}_{\mathbb{N}}$ is $\#^{\mathbb{N}}$.

This discussion is summarized in Theorem 4, which is Theorem 2 of [1].

Theorem 4. *Let $P :\subset \mathbb{N}^{\mathbb{N}} \rightrightarrows \mathbb{N}^{\mathbb{N}}$ be a problem, let $x \in \mathrm{dom}(P)$, and let α be a countable ordinal. The following are equivalent.*

1. *There is a set $A \subset \mathbb{N}^{\mathbb{N}}$ such that $P|_A$ is discontinuous, x is a local discontinuity of $P|_A$, and x has Cantor-Bendixson rank $\mathrm{rank}_A(x) \leq \alpha$.*
2. *$P \geq^*_{\mathrm{W}} \mathsf{ACC}^{\alpha}_{\mathbb{N}}$ via forward function Φ such that $\Phi(\#^{\mathbb{N}}) = x$.*
3. *Player 1 has a winning strategy in the rank-α discontinuity game for P with initial move x.*
4. *x is a rank-β discontinuity of P for some $\beta \leq \alpha$.*

If time permits, we will discuss applications of this framework to the thin set and achromatic Ramsey theorems, where ordinal-indexed color bounds yield problems whose exact discontinuity ranks can be computed.

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Antonio Nakid Cordero (Technische Universität Wien)

Effectively closed sets and enumeration reducibility

The connection between enumeration reducibility and the Scott topology on $\langle \mathcal{P}(\omega), \subseteq \rangle$—namely, that every continuous function is an enumeration operator relative to some enumeration oracle—motivates the study of the effectively closed sets of this space and their relationship to the enumeration degrees.

We focus on the question: *Which enumeration degrees bound the degree of a (maximal) point in every effectively closed set relative to a degree $\boldsymbol{a}$?* Our main result is a characterization of the cototal degrees that gives a partial answer to this question.

Orazio Nicolosi (Università degli Studi di Torino)

A game characterization for weak continuous reducibility

Joint work with Raphaël Carroy.

In this talk, I present a game that characterizes the weak continuous reducibility of functions between recursively presented metric spaces to functions between the Baire space ω^ω. This work uses the structure of continuous degrees in Effective Descriptive Set Theory to generalize part of the results presented in [2] to functions between recursively presented metric spaces.

Definition 1. Given X_f, Y_f, X_g, Y_g topological spaces and two functions $f : X_f \to Y_f$ and $g : X_g \to Y_g$, we say that f is *weakly continuous reducible* to g ($f \leq_w g$) if there exist a continuous function $\phi : X_f \to X_g$ and a partial continuous function $\psi : Y_g \times X_f \rightharpoonup Y_f$[1] such that $\forall x \in X_f (f(x) = \psi(g(\phi(x)), x))$.

Weak reducibility is also considered in Computable Analysis, where it is called *continuous Weihrauch reducibility.*

Definition 2. Let (X, d) be a separable metric space and $\boldsymbol{r} = (r_i)_{i\in\omega}$ an enumeration (possibly with repetitions) of a dense subset of X. We say that $\boldsymbol{r}$ is a *recursive presentation* of X if the relations on ω^3

$$P(i, j, k) \iff \mathrm{d}(r_i, r_j) \leq q_k$$
$$Q(i, j, k) \iff \mathrm{d}(r_i, r_j) < q_k$$

are computable. The structure $\mathcal{X} = (X, \mathrm{d}, \boldsymbol{r})$ is called *recursively presented metric space.* If moreover (X, d) is complete, then $\mathcal{X}$ is called *recursively presented Polish space.*

Given two functions $f : \omega^\omega \to \omega^\omega$ and $g : \mathcal{X}_g \to \mathcal{Y}_g$ where $\mathcal{X}_g$ and $\mathcal{Y}_g$ are recursively presented metric spaces, we define a two-player, zero-sum, perfect information game $G_M(f, g)$. This game characterizes weak reducibility between f and g in the sense that Player 2 has a winning strategy if and only if $f \leq_w g$. To get such a result, we modify the original definition of Lutz's game using the admissible representation $\rho_{\mathcal{X}} : \omega^\omega \rightharpoonup \mathcal{X}$ for recursively presented metric spaces $\mathcal{X} = (X, \mathrm{d}, \boldsymbol{r})$ defined as:

$$\rho_{\mathcal{X}}(p) = x \iff \mathrm{ran}(p) = \mathrm{N}_{\mathrm{base}}(x) := \{n \in \omega \mid x \in V_n^{\mathcal{X}}\}$$

where $V_i^{\mathcal{X}}$ is the i-th ball associated to the recursive presentation of $\mathcal{X}$ (i.e.,$V_i^{\mathcal{X}} = B_{q_{(i)_1}}(\boldsymbol{r}_{(i)_0}) = \{x \in X \mid d(x, \boldsymbol{r}_{(i)_0}) < q_{(i)_1}\}$[2]), and $(q_l)_{l\in\omega}$ is a fixed recursive enumeration of $\mathbb{Q}^+$. That is, we identify each point $x \in X$ with the elements of ω^ω that correspond to an enumeration of the neighborhood basis given by the balls with rational radius and centers in the dense subset $\boldsymbol{r}$.
We use such representation because it has nice (effective) topological properties, namely: it is Σ^0_1-recursive on its domain, open, surjective and with Polish fibers (see [5, Theorem 1]) and, moreover, satisfies the following property [1, Fact 2.4 (iv)] w.r.t. the structure of continuous degrees:

$$\forall x \in X \forall z \in \omega^\omega (x \leq_M z \iff \exists p \in \omega^\omega (\rho_{\mathcal{X}}(p) = x \wedge p \leq_T z))$$

Where $\leq_M$ denotes the representation reducibility defined as in [1, Definition 2.3] and [3, Definition 3.1]. This is a generalization of Turing reducibility to the context of recursively presented metric spaces.

Given a topological pointclass $\boldsymbol{\Gamma}$, for any function $f : X_f \to Y_f$ between topological spaces we write $f \in \mathrm{dec}(\boldsymbol{\Gamma})$ if there exists a partition $\{A_m\}_{m\in\omega}$ of X such that $f \restriction A_m$ is $\boldsymbol{\Gamma}$-measurable for all $m \in \omega$. In particular, $f \in \mathrm{dec}(\boldsymbol{\Sigma}^0_1)$ coincides with f being σ-continuous.

[1] We denote with the arrow $\rightharpoonup$ all partial functions.

[2] We denote with $()_0, ()_1 : \omega \to \omega$ the two functions which associate to the code of a pair the first and the second element (respectively).

Recall that the *Turing Jump* $J : \omega^\omega \to 2^\omega$ is the function which associates to each $x \in \omega^\omega$ the characteristic function corresponding to the x-relativized diagonalized Halting Problem $x' = \{e \in \omega \mid \varphi_e^x(e) \downarrow\}$. This function is $\mathbf{\Sigma}_2^0$-measurable, injective, and not σ-continuous. It can be iterated by composition, and the n-th iteration of the Turing Jump $J^{(n)} : \omega^\omega \to 2^\omega$ is $\mathbf{\Sigma}_{n+1}^0$-measurable but not in $\mathrm{dec}(\mathbf{\Sigma}_n^0)$.

Using the game G_M and the iterated Turing Jump, we extend the following dichotomy, which was originally proved in [2] for functions on the Baire space:

Theorem 3. *Given $\mathcal{X}$ and $\mathcal{Y}$ recursively presented metric spaces, and $g : X \to Y$ function such that $G_M(J^{(n)}, g)$ is determined, then either $g \in \mathrm{dec}(\mathbf{\Sigma}_n^0)$ or $J^{(n)} \leq_w g$.*

For the case $n = 1$, this dichotomy was originally proved by Solecki in [6] for Baire class 1 functions, although there it is formulated using the *Pawlikowski function* (which is $\equiv_w$-equivalent to the Turing Jump) and *topological embeddability*, which is a stronger form of reducibility between functions. Then, it was extended to all functions with analytic graphs by Pawlikowski and Sabok (see [4]).

In addition, we prove that $G_M(J^{(n)}, g)$ is always determined when g is a Borel function and is defined on a standard Borel space.

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Georgii Potapov (Royal Holloway, University of London)

Randomness deficiency with respect to a class

Given a finite string x and a probability distribution $\mathbf{P}$ assigning rational probabilities to a finite set of strings, we can define the randomness deficiency of x with respect to $\mathbf{P}$ as

$$\mathbf{d}(x \mid \mathbf{P}) = -\log \mathbf{P}(x) - \mathbf{K}(x \mid \mathbf{P}),$$

where $\mathbf{K}$ is the prefix Kolmogorov complexity, and the logarithm is base 2 ([1]).

For the case of an arbitrary computable metric space $\mathcal{X}$, randomness deficiency can be defined as the logarithm of a universal uniform randomness test $\mathbf{t}$: $\mathbf{d}(x \mid \mathbf{P}) = \log \mathbf{t}(x \mid \mathbf{P})$ ([2, 3]). A universal uniform test of randomness is the largest, up to a constant factor, lower semicomputable (l.s.c.) function taking values in $[0, +\infty]$ such that $\mathbb{E}_{X\sim\mathbf{P}}\mathbf{t}(X \mid \mathbf{P}) \leq 1$ for any probability distribution $\mathbf{P}$ on $\mathcal{X}$. Lower semicomputability of a function is defined in terms of effective openness of the function's subgraph, and thus does not put any computability constraints on the function's arguments.

For a given class $\mathcal{P}$ of probability distributions on $\mathcal{X}$, we can also consider the universal randomness tests with respect to $\mathcal{P}$ — defined as the largest l.s.c. functions $\mathbf{t}_{\mathcal{P}}$ such that $\mathbb{E}_{X\sim\mathbf{P}}\mathbf{t}_{\mathcal{P}}(X) \leq 1$ for all $\mathbf{P} \in \mathcal{P}$ — and the corresponding class randomness deficiency $\mathbf{d}_{\mathcal{P}}(x) = \log \mathbf{t}_{\mathcal{P}}(x)$. When the infimum $\inf_{\mathbf{P}\in\mathcal{P}} \mathbf{t}(\cdot \mid \mathbf{P})$ is equal to a l.s.c. function, then the class test for $\mathcal{P}$ exists and is equal to the infimum.

In [4] it is shown that $\inf_{\mathbf{P}\in\mathcal{P}} \mathbf{t}(\cdot \mid \mathbf{P})$ is l.s.c. when $\mathcal{P}$ is an effectively closed set, for example, if $\mathcal{P}$ is the class of binomial distributions $\{\mathrm{Bin}(n, p)\}_{p\in[0,1]}$ for a fixed n, or, similarly, the class of the Bernoulli distributions on the strings of a fixed length.

We will discuss computability of $\inf_{\mathcal{P}} \mathbf{t}$ and its explicit (in terms of uniform universal tests) form for some classes of probability distributions that are not topologically closed, such as the class $\{\mathcal{U}_{[0:n]}\}_{n\in\mathbb{N}}$ of uniform distributions on integer ranges $\{0, 1, \ldots, n\}$ and the class of Poisson distributions $\mathbf{Pois} = \{\mathbf{Pois}(\lambda)\}_{\lambda\in(0,\infty)}$. It can be proven, for example, that

$$\mathbf{d}_{\mathbf{Pois}}(n) = \inf_{\lambda} \mathbf{d}\big(n \mid \mathbf{Pois}(\lambda)\big) + O(1) = \frac{1}{2}\log n - \mathbf{K}(n \mid \lfloor\sqrt{n}\rfloor) + O(1).$$

Several other results of this type can be proven for different classes of probability distributions on natural numbers, as well as similar results for classes of continuous distributions, such as the class of normal distributions with fixed variance $\{\mathcal{N}(\mu, 1)\}_{\mu\in\mathbb{R}}$.

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Margarita Reyes García (Universidad Nacional Autónoma de México)

Approximating Kolmogorov complexity in fractal graphs using BDM: A methodological proposal

Self-similarity properties have been identified in several complex networks. In particular, Bunimovich and Skums [1] have shown that such self-similarity emerges from the properties of network communities and the relationships between them. Based on this idea, the analysis of the intersection patterns between these communities can be approached using the *box-covering* algorithm [2] in combination with an algorithmic complexity approach. This allows us to evaluate how the structural decomposition and organization at different scales determine the algorithmic complexity of the network. Additionally, using a procedure inspired by the randomization process associated with the Watts-Strogatz model [3], it is possible to analyze the robustness of the inter-community relationship under different degrees of network randomization.

However, the main challenge is that Kolmogorov complexity is a non-computable measure. To overcome this limitation, Héctor Zenil [4] formulated the Block Decomposition Method (BDM). This method (in combination with the Coding Theorem Method [5]) allows approximating the algorithmic complexity of a binary sequence by decomposing it into smaller parts whose complexity is previously known. BDM can be extended to matrices, making it possible to study richer structures, such as complex networks.

Since this work is currently in its initial phase, this presentation will focus on showing the relevance of the structures to be explored and, after reviewing the literature, on justifying the proposed methodology.

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Dino Rossegger (Technische Universität Wien)

A topological highness notion

We isolate a class of continuous functions from Polish spaces to Cantor space, called high functions. We prove characterizations of this class similar to those of high degrees in degree theory, give an application to the theory of topological realizations of countable Borel equivalence relations, and discuss ongoing research as well as highlight avenues for future research opened by this line of investigation.

This is work in progress. A write-up of the main characterization including detailed proofs is in [Ros].

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Isabella Scott (University of Wisconsin–Madison)

Comparing presentations of Polish spaces

Computable-topological reducibility (or ct-reducibility) provides a framework for measuring the effective content of different computable presentations of the same Polish space. Different spaces may have wildly different degree structures, which can reflect the topological properties of the original space. Further, in Cantor space, the ambient dimension of the presentation affects the "location" of its ct-degree. We will discuss the extent to which topological properties are reflected by ct-degrees, as well as pathologies that show up.

This is joint work, in various projects, with Noam Greenberg, Steffen Lempp, Sasha Melnikov, Joe Miller, Mariya Soskova, and Dan Turetsky.

Yudai Suzuki (National Institute of Technology, Oyama College)

Generalizations of the second incompleteness theorem and its surroundings

In this talk, we introduce two versions of Gödel's second incompleteness theorem. We revisit the second incompleteness theorem from two perspectives.

One perspective is the relationship between the definability complexity of a theory and the unprovability of its soundness. From the perspective of soundness, the second incompleteness theorem can be stated as 'any Σ_1-definable Π_1-sound extension of $I\Sigma_1$ does not prove its own Π_1-soundness.' In this talk, we consider the relationship between Σ_n-definability and Π_k-soundness for $n, k \in \omega$. We also consider the logical strength of Craig's trick that correlates Σ_n-definability and Π_{n-1}-definability.

The other perspective is semantic incompleteness. The second incompleteness theorem may be seen as the unprovability of the existence of a model. It is known that 'model' is replaced by 'ω-model' [1] or 'β_n-model' [2,3] for some theories of second-order arithmetic. We give a new and unified proof of the ω-model and β_n-model versions of the incompleteness theorem.

This talk is based on joint work with Keita Yokoyama [4].

This work is partially supported by JSPS KAKENHI grant number JP26K17028.

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Keita Yokoyama (Tohoku University)

Nonstandard arguments in reverse mathematics (in memory of Kazuyuki Tanaka)

Kazuyuki Tanaka's self-embedding theorem for WKL_0, together with the nonstandard method built upon it, stands as a milestone in the development of reverse mathematics [1]. Using this method, Tanaka developed basic nonstandard analysis within WKL_0. Tanaka and Yamazaki then used the method to construct Haar measure on compact groups [2]. It was later developed further by the Tohoku group. For example, a nonstandard proof of the Jordan curve theorem can be carried out in WKL_0, while a nonstandard proof of the Riemann mapping theorem can be carried out in ACA_0. For arguments of the latter kind, a formal system for nonstandard analysis over ACA_0, including a Σ^1_1 transfer principle, was also introduced. Related nonstandard approaches connected with reverse mathematics have been studied by H. Jerome Keisler, Sam Sanders, and others. In the first part of this talk, I will give an overview of these results and their background.

As a recent example of the use of nonstandard methods in reverse mathematics, I will present joint work with Aguilera and Kouptchinsky on the measurability of analytic sets [3]. We show that analytic Lebesgue regularity is equivalent to Σ^1_1-IND over ATR_0, while the full measurability statement, which asserts the existence of the measure as a real number, is equivalent to Π^1_1-CA_0.

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