Workshop on Computability Theory 2011, part 2
Thematic Day in Computability

Centre de Recerca Matemàtica - Barcelona, Spain - July 17, 2011

Schedule

9:30	Morning coffee
9:50	Margarita Leontyeva
11:00	Klaus Ambos-Spies
12:00	Andrey Morozov
1:00	Lunch
2:30	Valentina Harizonov
3:30	Paul Shafer
4:30	Andrey Frolov

Abstracts

Klaus Ambos-Spies, University of Heidelberg

How much does a typical set know about exponential time?

In our talk we present some new results and survey the results in the literature on the question of which part of the deterministic exponential time class E = DTIME($2^{O(n)}$) can be reduced to a typical set by a polynomial-time many-one reduction. We consider typical sets in general but also typical computable sets, typical sets in the polynomial exponential time class EXP = DTIME($2^{poly(n)}$), and typical sets in E where typicalness is correspondingly defined in terms of measure, computable measure, and resource-bounded measures (or, equivalently, in terms of the corresponding levels of randomness). In order to measure the sizes of the parts of E which can be reduced we use the notions of completeness (= all of E), Lutz's measure completeness (= a nonnegligible part of E in the sense of resource-bounded measure), and the more recent notions of strong nontriviality and nontriviality introduced by Ambos-Spies and Bakibayev (= almost everywhere resp. infinitely often complex sets from all levels DTIME($2^{k \cdot n}$), $k \geq 1$, of E). As we will show, the results will depend on the base class from which the typical set is chosen. (Joint work with Timur Bakibayev)

Andrey Frolov, Kazan State University

Computable linear orderings

First, I will give a wide class $\mathcal{A}$ of types of linear orderings such that if a low linear ordering has a type $\mathcal{A}$ then the ordering has a computable presentation. In the second part of my talk I will give properties of the successor relation of computable linear orderings. In particular, I will talk about the closure upwards of the spectrum of the successor relation in the enumerable degrees.

Valentna S. Harizanov, George Washington University

When Orders on a Group Form the Cantor Set

In recent years, the theory of orders on groups has become an important tool in understanding the geometric properties of 3-dimensional manifolds. A group is left-orderable if there is a linear ordering of its domain, which is left-invariant with respect to the operation. If the ordering is also right-invariant, then the group is bi-orderable. For a group G, Sikora defined a natural topology on the space of its left orders. He showed that if G is countable, then this space is compact, metrizable, and totally disconnected. Furthermore, he showed that for a computable torsion-free abelian group of a finite rank greater than 1, the space of bi-orders is homeomorphic to the Cantor set. We further investigate when the space of left orders or bi-orders on familiar computable groups is homeomorphic to the Cantor set and how this topological property relates to the computability theoretic complexity of orders. While there are countable groups with infinitely countably many bi-orders, the space of left orders of a countable orderable group is either finite or contains a homeomorphic copy of the Cantor set.

Margarita Leontyeva, Novosibirsk State University

Boolean algebras and their strongly computable representations Abstract:

Boolean algebras are classical object that appears in different parts of mathematics and performs to be popular subject of investigation for many mathematicians for more then one and a half century. This talk is going to arise within computability theory concerning Boolean algebras and primarily the existence of their strongly computable representations in terms of computable sequence of predicates on Boolean algebra introduced by Yurii Ershov in 1964. This problem has been investigated for more then 40 years by S. Goncharov, P. Alaev, S. Odintsov, V. Vlasov and others. We will talk about these results starting with the work of Ershov till the most recent statements and give the current situation in the field, including open problems.

Andrey S. Morozov, Novosibirsk State University

On some Σ–presentations of the field of reals

We prove that any two Σ–presentations of the ordered field of the reals R over $HF(R)$ whose basic set is a subset of R are Σ–isomorphic. It follows that for a series of functions $f : R \to R$ (for instance, for exp, sin, cos, ln), the structure $\langle R, +, \times, <, 0, 1, f\rangle$ fails to have Σ–presentations over $HF(R)$ whose basic set is contained in R.

Paul Shafer, Appalachian State University

Complexity in the Medvedev degrees of Π^0_1 classes

We first give an overview of the Medvedev and Muchnik degrees and of various complexity results for these structures. We continue by discussing in detail two new complexity results. The first result is that the first-order theory of the Medvedev degrees of Π^0_1 classes is recursively isomorphic to the first-order theory of true arithmetic. The second result is that an oracle computes a presentation of the Medvedev degrees of Π^0_1 classes if and only if the oracle computes $\emptyset'''$.