Workshop on Computability Theory 2019
Leeds, UK — July 22 & 23, 2019

Schedule

The workshop is held in the Chemistry Building on Woodhouse Lane. The talks are in Lecture Theatre D, and the coffee breaks are in the Chapman Lounge.

Monday, July 22

Before 9:50am:	Coffee available! (Chapman Lounge)
9:50am – 10:00am:	Welcome to Leeds!
10:00am – 10:30am:	Alberto Marcone *Reorientations of directed graphs*
10:40am – 11:10am:	Emanuele Frittaion *The primitive recursive ω-rule*
11:20am – 11:50am:	Sabrina Ouazzani *How to compute with an infinite time Turing machine?*
12:00pm – 2:00pm:	Lunch!
2:00pm – 2:30pm:	Jun Le Goh *Inseparable Π^1_1 sets*
2:40pm – 3:10pm:	Mars Yamaleev *The Ershov hierarchy and the structures of many-one and Turing degrees*
3:20pm – 3:50pm:	Coffee break! (Chapman Lounge)
3:50pm – 4:20pm:	David Fernández-Duque *Predicatively unprovable termination of the Ackermannian Goodstein process*
4:30pm – 5:00pm:	Keita Yokoyama *Approaching the first-order part of Ramsey's theorem for pairs and two colors*

Tuesday, July 23

Before 10:00am:	Coffee available! (Chapman Lounge)
10:00am – 10:30am:	Russell Miller *Computable Structure Theory for Models of* **ZFC**
10:40am – 11:10am:	Johanna Franklin *Lowness for paths*
11:20am – 11:50am:	Arno Pauly *Finding points from positive information*
12:00pm – 2:00pm:	Lunch!
2:00pm – 2:30pm:	Elvira Mayordomo *The return trip to classical fractal theory from effective fractal dimension*
2:40pm – 3:10pm:	Ethan McCarthy *Cototal enumeration degrees in symbolic dynamics*
3:20pm – 3:50pm:	Coffee break! (Chapman Lounge)
3:50pm – 4:20pm:	Dino Rossegger *Degree spectra with respect to equivalence relations*
4:30pm – 5:00pm:	Julia F. Knight *Coding of structures so that decoding is effective*

Abstracts (alphabetical by author)

David Fernández-Duque (Ghent University)

Predicatively unprovable termination of the Ackermannian Goodstein process

The classical Goodstein process gives rise to long but finite sequences of natural numbers whose termination is not provable in Peano arithmetic. In this talk we consider a variant based on the Ackermann function. We show that Ackermannian Goodstein sequences eventually terminate, but this fact is not provable using predicative means.

This is joint work with Toshiyasu Arai, Stanley Wainer, and Andreas Weiermann.

Johanna Franklin (Hofstra University)

Lowness for paths

We define lowness notions for Π^0_1 classes in Cantor space and Baire space. We demonstrate that these notions coincide with each other as well as with lowness for isomorphism.

This work is joint with Dan Turetsky.

Emanuele Frittaion (University of Leeds)

The primitive recursive ω-rule

Various constructive restrictions of the ω-rule

$$\text{From } A(0), A(1), \ldots, A(n), \ldots \text{ Infer } \forall x A(x)$$

have been considered for proof theoretic purposes (in, e.g., [1–3]). I will discuss coding aspects and related completeness-complexity issues regarding the so-called primitive recursive ω-rule.

[1] Solomon Feferman. Transfinite recursive progressions of axiomatic theories. *The Journal of Symbolic Logic*, 27(3):259–316, 1962.

[2] Helmut Schwichtenberg. Proof theory: Some applications of cut-elimination. In *Studies in Logic and the Foundations of Mathematics*, volume 90, pages 867–895. Elsevier, 1977.

[3] Ulf R. Schmerl. Iterated reflection principles and the ω-rule. *The Journal of Symbolic Logic*, 47(4):721–733, 1982.

Jun Le Goh (Cornell University)

Inseparable Π^1_1 sets

We investigate analogues of the theory of effectively inseparable pairs of recursively enumerable sets to Π^1_1 sets of numbers and Π^1_1 sets of reals. These are used to derive completeness results, such as an unpublished result of Harrington about jump hierarchies.

Julia F. Knight (University of Notre Dame)

Coding of structures so that decoding is effective

The notion of Turing computable embedding captures the idea of coding structures from one class K in those from another class K'. It is interesting when there is a uniform effective procedure for decoding, and also when decoding is very difficult. Suppose that $K \leq_{tc} K'$ via Φ, so that for each input structure $\mathcal{A} \in K$, $\mathcal{A}$ is coded in the output structure $\Phi(\mathcal{A})$. Uniform Medvedev reductions of the input structures to the output structures captures the idea of effective decoding. Harrison-Trainor, Melnikov, R. Miller, and Montalbán introduced a stronger notion of effective interpretation, which involves computable Σ_1 formulas with no parameters. There are known embeddings showing that the following classes lie on top under Turing computable embeddings: graphs, fields, 2-step nilpotent groups, and linear orderings. Some of these embeddings come with uniform effective interpretations, so that we have uniform effective decoding. I will focus on some examples in which the original interpretations involve parameters, but these can be removed. The work is joint with Andrey Morozov, Russell Miller, Alexandra Soskova, Valentina Harizanov, Wesley Calvert, Rachael Alvir, Rose Weisshaar, and Grant Goodman.

Alberto Marcone (University of Udine)

Reorientations of directed graphs

A directed graph (V, E) is pseudo-transitive if whenever $(a, b) \in E$ and $(b, c) \in E$, we have either $(a, c) \in E$ or $(c, a) \in E$. (V, E) is transitive if, under the same hypothesis, $(a, c) \in E$ always occurs.

A reorientation E' of (V, E) is obtained by turning some of the edges of the directed graph around, i.e. for any a the sets of neighbors of a with respect to E and E' are the same. In 1962, Alain Ghouila-Houri proved that every pseudo-transitive directed graph has a transitive reorientation. Actually the original proof dealt only with finite graphs, but a straightforward compactness argument extends the result to graphs of every cardinality. This implies that WKL_0 suffices to prove the theorem for countable graphs, and $\mathsf{C}_{2^\mathbb{N}}$ computes (in the sense of Weihrauch reducibility) the corresponding multi-valued function.

After spending quite some time trying to obtain reversals, our research led to a new algorithmic proof of Ghouila-Houri's theorem, showing that neither WKL_0 nor $\mathsf{C}_{2^\mathbb{N}}$ are necessary.

This is joint work with Marta Fiori Carones.

Elvira Mayordomo (University of Zaragoza)

The return trip to classical fractal theory from effective fractal dimension

J. Lutz and N. Lutz (2017) have recently proven a point-to-set principle for Euclidean and Cantor spaces. This result is a characterization of classical Hausdorff dimension in terms of relativized effective dimension. This implies that geometric measure results regarding Hausdorff dimension can be shown using only effective methods. Several interesting classical results have already been proven using this principle.

In this talk I will present the point-to-set principle in Euclidean space and explain in detail how a Kolmogorov complexity proof (N. Lutz 2017) improves a well-known classical result.

Next I will generalize this approach to any separable metric space and show examples of information theory proofs of fractal dimension results in these spaces, including a new result that generalizes known results on the hyperspace.

Ethan McCarthy (University of Florida)

Cototal enumeration degrees in symbolic dynamics

The enumeration degrees measure the computability-theoretic difficulty of enumerating sets of natural numbers. In contrast to the Turing degrees, a set and its complement need not have comparable enumeration degree. The enumeration degrees of sets which are enumeration-above their complement, called total degrees, give an embedded copy of the Turing degrees in the enumeration degrees. Dual to the total degrees, the cototal degrees are the degrees of sets which are enumeration below their own complements. We will explore how the cototal degrees capture the computability-theoretic complexity of objects from various areas of mathematics, in particular the minimal subshifts of symbolic dynamics.

Russell Miller (Queens College, City University of New York)

Computable Structure Theory for Models of **ZFC**

The process of forcing has become second nature for many set theorists. We examine the extent to which it is actually an effective process, in terms of computable structure theory. What information is assumed, and how much of it is actually necessary to build a generic extension $M[G]$ of an (externally countable) model M of ZFC, given a forcing notion within M? To what extent can this process be performed algorithmically?

This is joint work with Joel David Hamkins and Kameryn Williams.

Sabrina Ouazzani (École Polytechnique)

How to compute with an infinite time Turing machine?

In this talk, we present infinite time Turing machines (ITTM), from the original definition of the model to more specific properties and developments.

We will introduce some properties of the ITTM-computable ordinals. In particular, we will study gaps in ordinal computation times, that is to say, ordinal times at which no infinite time program halts.

Then we will end by a presentation of some new applications of the model.

Arno Pauly (Swansea University)

Finding points from positive information

If we know which open sets contain solutions, can we find a solution? We show that for countably-based spaces, a positive answer is equivalent to the space being quasi-Polish. For non-countably based coPolish spaces, the answer is no – but how complicated the Weihrauch degree of the problem is depends on whether or not the space is Frechet-Urysohn.

This is joint work with Matthew de Brecht and Matthias Schröder. A preprint is available here: `https://arxiv.org/abs/1902.05926`.

Dino Rossegger (Vienna University of Technology)

Degree spectra with respect to equivalence relations

The study of degree spectra of structures is a central topic in computable structure theory. Recently, researchers generalized this notion to facilitate the study of the Turing degrees of structures equivalent under a given equivalence relation. Given a structure $\mathcal{S}$, this notion is called the degree spectrum of $\mathcal{S}$ with respect to the equivalence relation E, or simply the *E-spectrum* of $\mathcal{S}$. Under this new notion the classical degree spectrum would be called isomorphism spectrum, but also degree spectra with respect to several other equivalence relations have been studied. Examples of such equivalence relations are elementary equivalence, bi-embeddability, and elementary bi-embeddability.

Two goals of this approach are, first, to exhibit families of degrees which can, or can not be realized as degree spectra under a given equivalence relation. Second, to distinguish equivalence relations based on the families of degrees they can realize as degree spectra.

In this talk we discuss recent advances towards these goals. We present new bi-embeddability and elementary bi-embeddability spectra and discuss a new approach towards the second goal which aims to connect the complexity of the equivalence relation and the families of degrees realizable as degree spectra with respect to it.

Mars Yamaleev (Kazan Federal University)

The Ershov hierarchy and the structures of many-one and Turing degrees

A set $A \subset \omega$ is called n-c.e. if it has a computable approximation such that for each element we start from zero and have at most n mind changes.

In the first part of the talk we will discuss our recent joint work with Ng Keng Meng (Selwyn), where we obtained several results about model-theoretic properties of the structures of m-degrees from the finite levels of the Ershov hierarchy. In particular, we obtained that in the partial ordering of n-c.e. m-degrees one can define k-c.e. m-degrees for all $n > k \geq 1$.

The second part of the talk will be devoted to the Turing degrees. The well-known Cooper's theorem states that there is a proper n-c.e. Turing degree for any natural $n > 0$, moreover, due to Selivanov, the result can be generalized to the transfinite levels of the Ershov hierarchy. In recent work with Selivanov V.L., we obtained that Cooper's theorem can be extended even to almost all levels below Σ_{ω^ω} in the fine hierarchy. Intuitively, this hierarchy is based on the Ershov hierarchy and allows very subtle distinctions of sets from the arithmetical hierarchy. Very recently, in joint work with Melnikov A.G. and Selivanov V.L., we were able to show that some work can be done even beyond the Σ_{ω^ω} level, where we faced Σ^0_3 sets in oracles of the Turing functionals in a computable construction.

The author is supported by RFBR project No. 18-31-00420.

Keita Yokoyama (Japan Advanced Institute of Science and Technology)

Approaching the first-order part of Ramsey's theorem for pairs and two colors

In this talk, we will overview recent approaches to calibrate the first-order strength of Ramsey's theorem for pairs and two colors (RT^2_2). Cholak/Jockusch/Slaman showed that the Π^1_1-part of $\mathsf{WKL}_0 + \mathsf{RT}^2_2$ is weaker than $\mathsf{I}\Sigma^0_2$ and asked whether it is Π^1_1-conservative over $\mathsf{B}\Sigma^0_2$ or not. Since then, there have been a large number of studies to answer this question with various approaches. Here, we will see a characterization of the Π^0_n-part of $\mathsf{WKL}_0 + \mathsf{RT}^2_2$ and a new candidate for a counterexample of Π^1_1-conservation provided by recent studies of models of arithmetic.