Minimal Pairs in the Generic Degrees

Denis R. Hirschfeldt

"What one knows, the other does not."

— Jean Froissart

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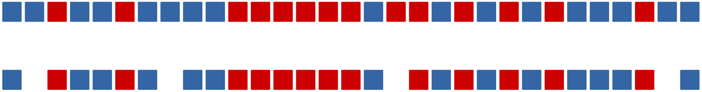

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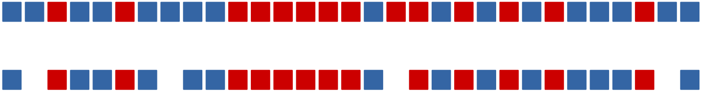

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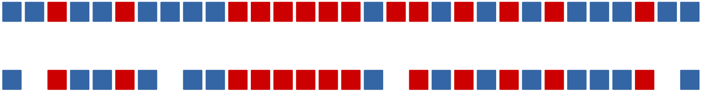

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This notion was introduced by Kapovich, Myasnikov, Schupp, and Shpilrain. It was later studied by Jockusch and Schupp.

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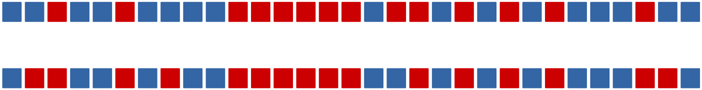

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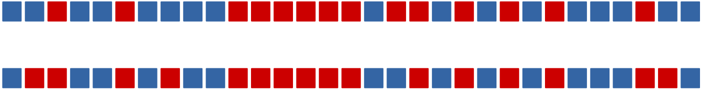

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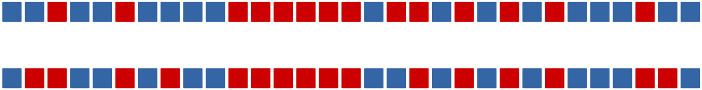

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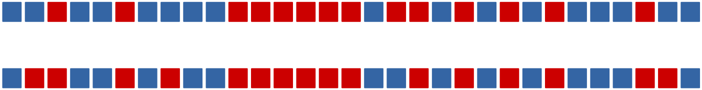

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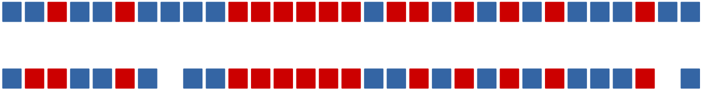

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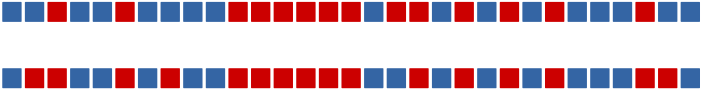

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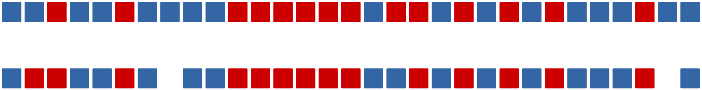

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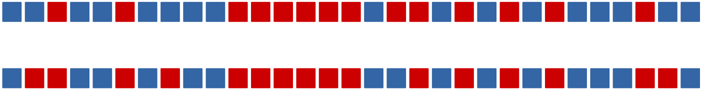

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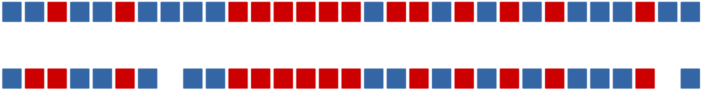

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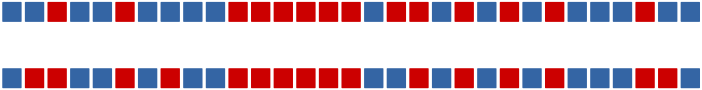

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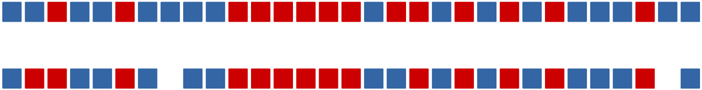

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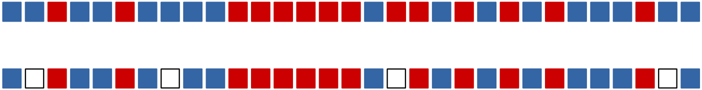

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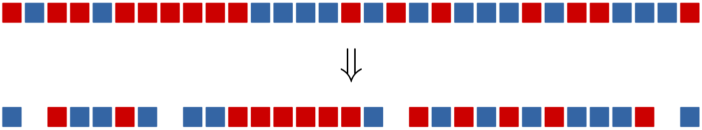

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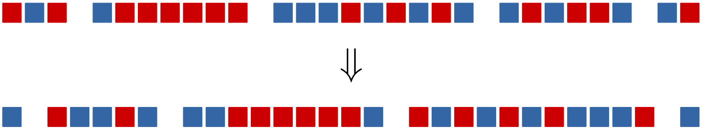

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↓

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↙ ↘

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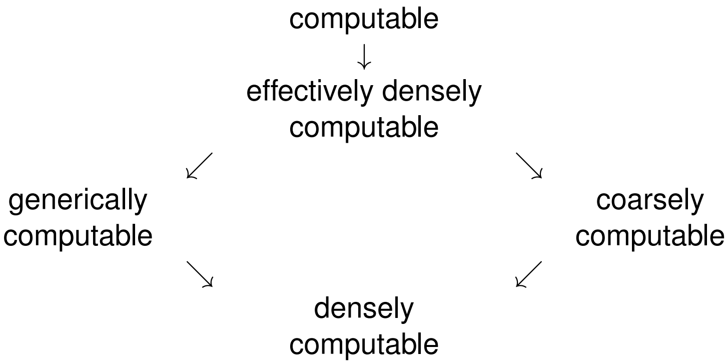

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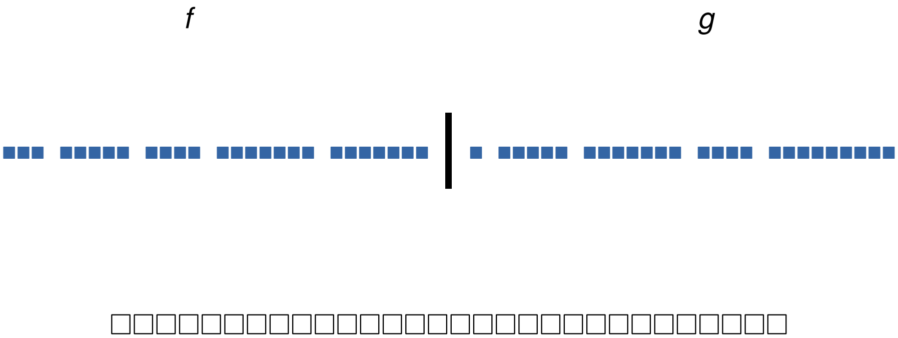
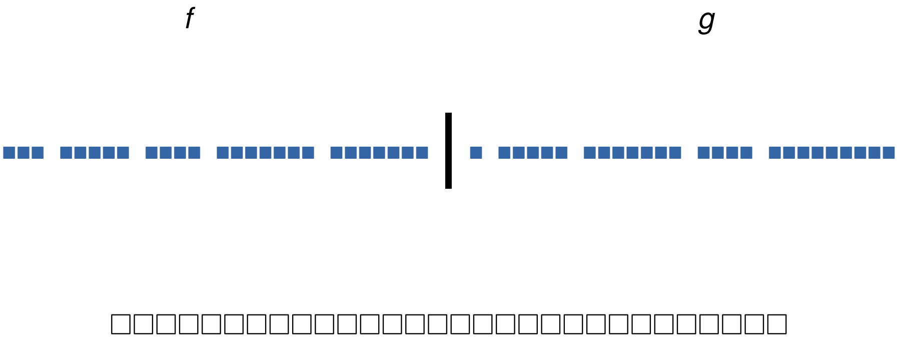
For enumeration operators Φ and Ψ, if Φ^f and Ψ^g have domains of density 1 and agree where both are defined, then we build a partial computable h s.t. $\operatorname{dom} h$ has density 1, and h agrees with at least one of Φ^f and Ψ^g where h is defined.

f *g*



We approximate *f* and *g* to be generic descriptions of *X* and *Y*, respectively, and ensure that *X* and *Y* are not generically computable.



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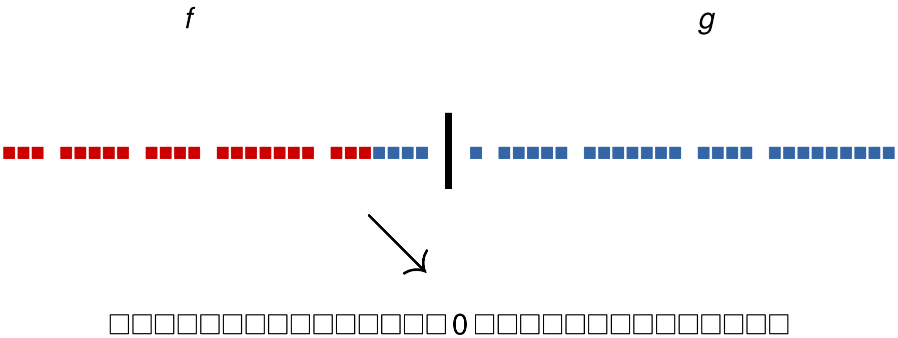

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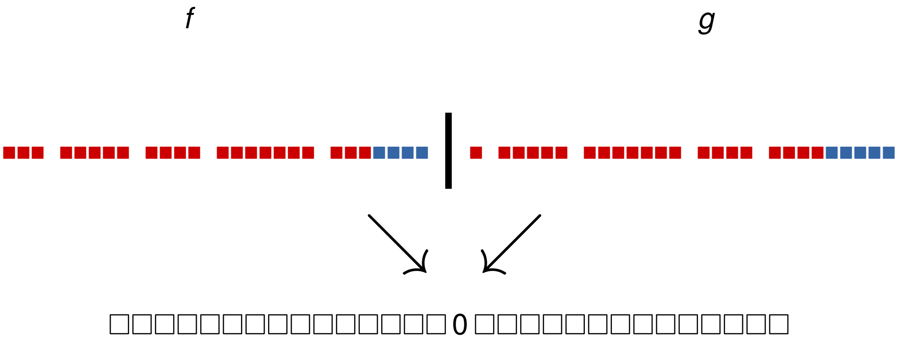

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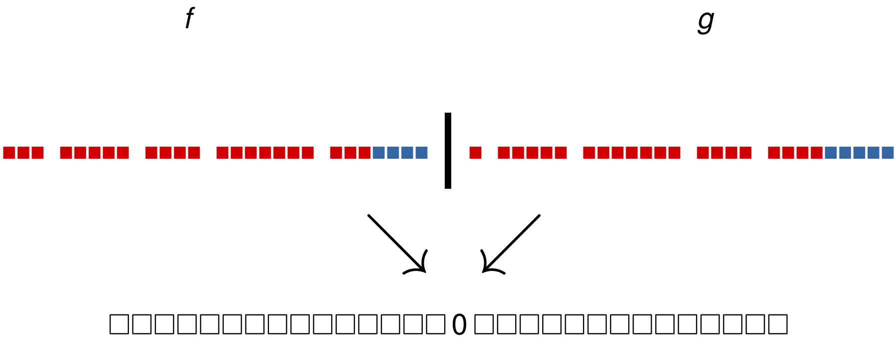
We define $h(n) = 0$.



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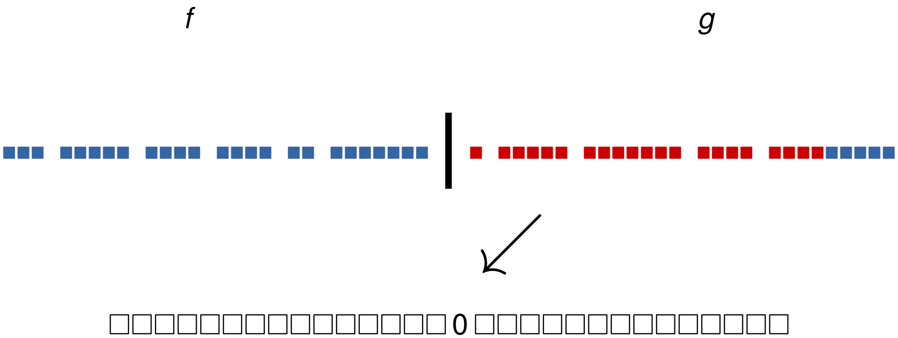

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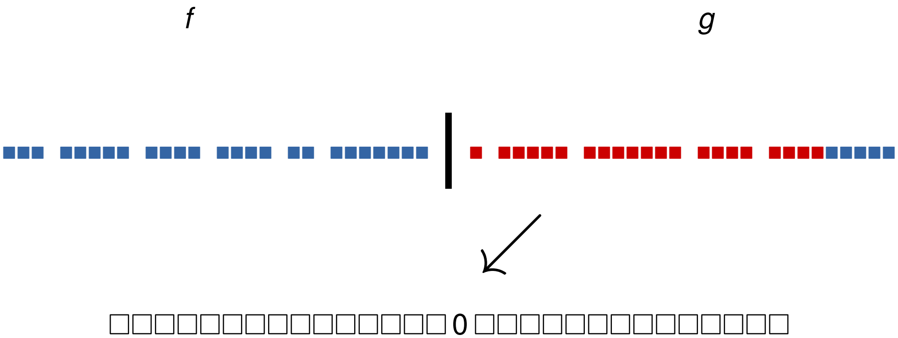

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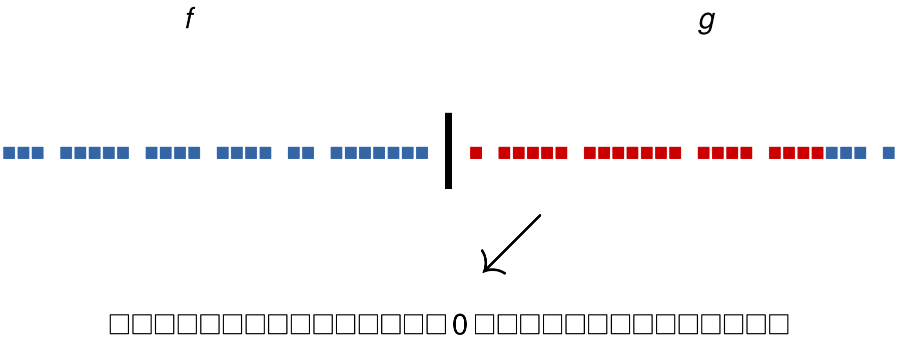

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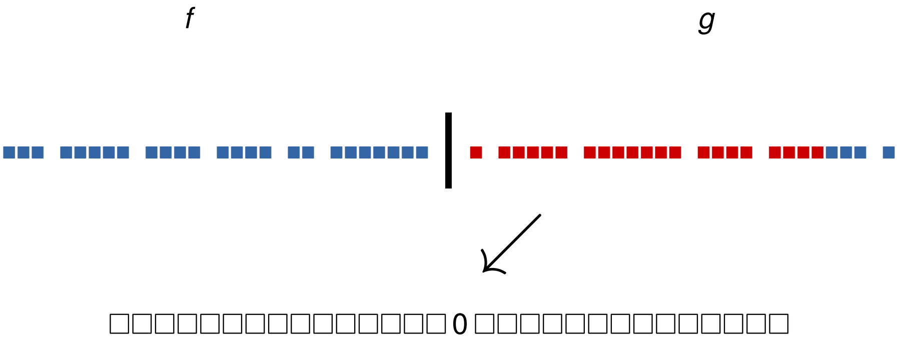

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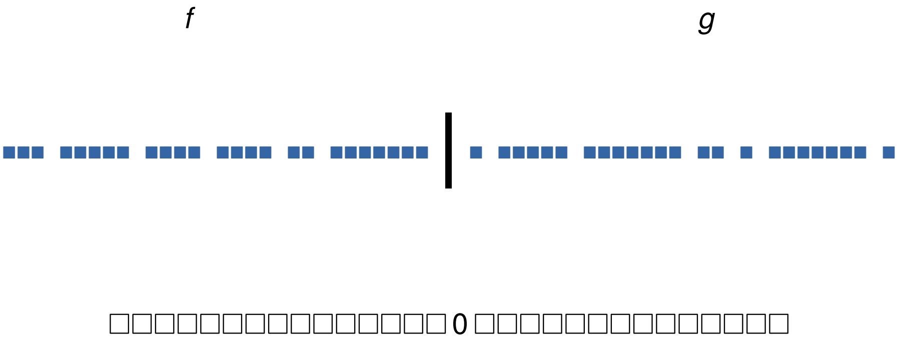

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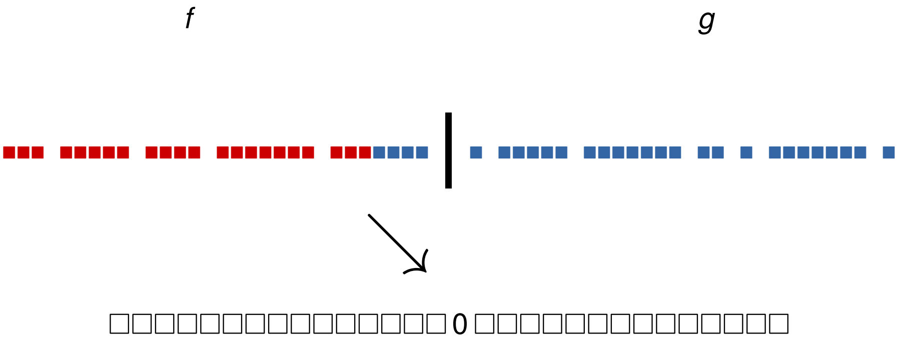

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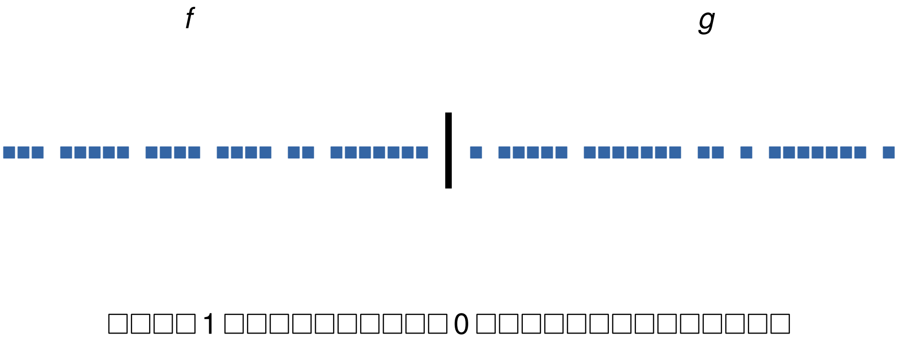

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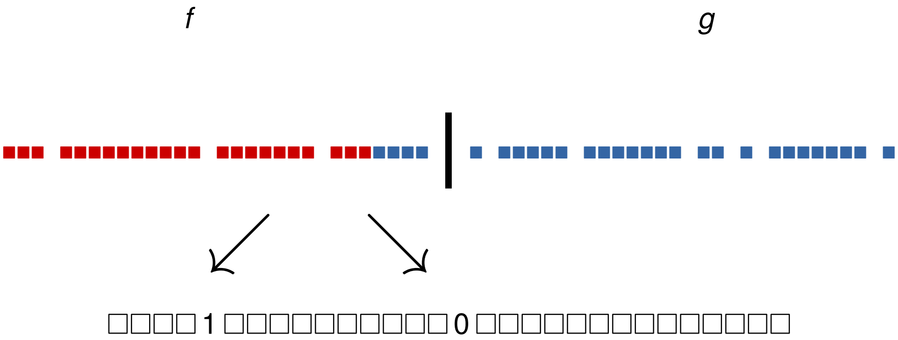

If a stronger requirement violates the use on the g side, we would like to restore the f side.

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Thm (Hirschfeldt, Jockusch, Kuyper, and Schupp; Astor, Hirschfeldt, and Jockusch). Nontrivial upper cones are null for relative coarse, generic, dense, and effective dense computability.

Open Questions

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Must minimal pairs in the generic degrees involve coarsely (or at least densely) computable sets?

Are there measure-1 many minimal pairs in the generic degrees?

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Are there Δ^0_2 sets whose coarse or dense degrees form a minimal pair?

Open Questions

(Jockusch and Schupp; Igusa)

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Does every generic degree bound a nonzero generic degree containing a density-1 set?

Open Questions

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Are there minimal pairs in the effective dense degrees?

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Are there minimal pairs for relative effective dense computability?

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Are there minimal pairs for relative effective dense computability?

Thm (Astor, Hirschfeldt, and Jockusch). Nontrivial upper cones in the generic degrees are meager.

Are nontrivial upper cones in the coarse, dense, and effective dense degrees meager?

There are natural embeddings of the Turing degrees into each of our degree structures. **(Jockusch and Schupp; Dzhafarov and Igusa; Hirschfeldt, Jockusch, Kuyper, and Schupp; Astor, Hirschfeldt, and Jockusch)**:

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Let $\mathcal{R}(A) = \{2^n k : n \in A \wedge k \text{ odd}\}$.

Let $J_n = [2^n, 2^{n+1})$ and $\mathcal{E}(A) = \bigcup_{m \in \mathcal{R}(A)} J_m$.

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$\mathcal{E}$ induces such embeddings.

For any of our degree structures, a nonzero degree is **quasiminimal** if it is not above any nonzero degree in the image of this embedding.

We would expect “typical” sets to have quasiminimal degrees.

Thm (Hirschfeldt, Jockusch, Kuyper, and Schupp; Cholak and Igusa; Cholak, Hirschfeldt, and Igusa; Astor, Hirschfeldt, and Jockusch).

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Every 1-generic is quasiminimal in all of our degree structures.

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Every weakly 2-random is quasiminimal in all of our degree structures.

Every 1-random is quasiminimal in the uniform generic, coarse, and effective dense degrees.

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Every 1-generic is quasiminimal in all of our degree structures.

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There are 1-randoms that are not quasiminimal in the nonuniform generic, coarse, dense, and effective dense degrees.

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Every 1-random is quasiminimal in the uniform generic, coarse, and effective dense degrees.

There are 1-randoms that are not quasiminimal in the nonuniform generic, coarse, dense, and effective dense degrees.

Open Question. Is every 1-random quasiminimal in the uniform dense degrees?

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